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A Multi-Objective Fuzzy Linear Programming Model for Cash Flow Management

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Abstract
Although significant research work has been conducted on cash flow forecast, planning, and management, the objective is constantly the maximization of profit/final cash balance, or minimization of total project cost. This paper presents a multi-objective fuzzy linear programming model (FLP) for resolving the optimization problem of three conflicting objectives: final cash balance, cost of money, and initial cash balance. The proposed model depends on Jiang et al. (2011) Model. In the new formulation, both the advanced payment and delay of owner's progress payment one period were considered. Literature concerned with cash flow studies and models for construction projects was reviewed. Fuzzy linear programming applications in literature was presented and it's concept was then described. Jiang et al. (2011) Model is presented. The proposed model development was then presented. The proposed model was validated using an example project. An optimization of each individual objective was performed with a linear programming (LP) software (Lindo) that gave the upper and lower bounds for the multi-objective analysis. Fuzzy linear programming was then applied to optimize the solution. Four cases are considered: considering advanced payment and delay of owner's progress payment one period simultaneously, then separately, and neglecting advanced payment and delay of owner's progress payment. Penalty of delayed payment have been also considered. Analysis of the results revealed that the model is an effective decision making tool to be used by industry practitioners with reasonable accuracy.
Key words: fuzzy linear programming; final cash balance; cost of money; initial cash balance; advanced payment; delay of payment.

I. Introduction

Contractors cannot survive in the competitive industry without effective cash flow management (Liu et.al.)[1]. Studies and investigations have shown that lack of liquidity is a major problem causing construction project failure (Al-Issa and Zayed)[2]. Cash is the most important resource for a construction company, because more companies become bankrupt due to lack of liquidity for supporting their day-to-day activities, than because of inadequate management of other resources (Singh and Lakanathan)[3]. Many construction projects have negative cash flows until the very end of construction when the final payment is received or advanced payment is received before starting the project (Jiang et al.)[4]. However, there is no project in progress that is in complete accordance with initial planning. This does not mean that there is no need for planning of the cash flow. A cash flow estimate that includes the uncertainties of the construction business will be more precise than a cash flow forecast based on the pre-estimate or estimate stage (Park)[5]. Cash Flow at the project level consists of a complete history of all cash disbursement, cash shortage, loans, cost of money, and all earnings received as a result of project execution (Jiang et al.)[4]. A firm with higher cash flow variability increases the level of expected external financing costs, which incurs high cost of money and accordingly high project cost. Jiang et al.[4] presented a Pareto optimality efficiency network model that considers the typical instruments and constraints of the financial market, including earnings from depositing excess cash, long term and short term loans from banks and minimum cash reserves for a project. Although this model considers a good deal of external and internal variables, it is still a limited representation of the complex real world of the construction management environment. The model does not consider the effect of important factors such as: advanced payment and delay of the client's progress payment which are major issues in project cash flow planning and management. These issues are considered in the current research.

This paper presents a multi-objective fuzzy linear programming model with the objective of maximizing final cash balance, minimizing total cost of money, and minimizing initial capital simultaneously. The proposed model depends on the model presented by Jiang et al.[4] with modifications in the development. In the new formulation, the proposed model considers the effect of both advanced payment and delay of the client's progress payment. Also, the penalty of delayed payment is considered. Thus, this research presents an advancement in both the development of the model and in the tool used for

solving the problem of optimization. Hereinafter, delay of owner's progress payment one period will be referred to as delay of payment for ease reference.
The methodology of this research passes through various steps. The second section is devoted to the review of previous studies and models of cash flow management. The third and fourth sections present fuzzy linear programming applications in literature and description of multi-objective fuzzy linear programming concept, respectively. The fifth section explains Jiang et al. model [4]. The proposed modifications to Jiang et al. model [4] and the new formulation are then presented. An illustrative example problem is then presented to validate the model considering different cases of dealing with advanced payment and delay of payment. Analysis of the results of the example problem helps indicate the contributions of the proposed model. Conclusions are given in the last section.

II. Cash Flow Studies and Models

Probably the earliest work was conducted by Hardy[6] examining "S" curves of 25 different project types; Mackay's [7] performed sensitivity analysis of net cash flow profiles to different value curves implied that either net cash flow curves conform to predictable patterns or that they were sensitive to the selection of payment delays. Bromilow and Henderson [8] developed values of 'S' curves for four categories of project type. Balkau [9] generated a value of "S" curve model. Reinschmidt and Frank [10] proposed a model for cash flow forecasting in the early planning stage of a project. Drake[11] and Hudson [12] developed polynomial models. Ashley and Teicholz [13] suggested a cash flow forecast based on detailed methods for cost flow. They classified direct cost by a number of cost categories such as labor, material etc., which are specified as percentages of the total cost. In their work, Fondahl and Bacarreza [14] applied three cost curves to their school project. Curve 1 is based on the assumption that rate of expenditure will be uniform over the project duration. Curve 2 assumes that only 25% of the total cost is incurred during the first half of the project duration and the remaining 75% in the second half. Curve 3 assumes that 75% of the total cost is incurred in the first half of project duration. Gates and Scarpa [15] described a simple approximation method for developing cash flow analysis income and expenses, surplus and deficit, as a function of time-over the life of the project. Kaka and Price [16] improved the accuracy of cash flow forecast by using commitment curves. Kaka and Price [17] developed a series of typical S-curves. They identified some of the risk factors affecting cash flow profile. These include estimating error, tendering strategies, cost and duration. Hsu [18] established statistic models to forecast control or assess of construction project cash flow by S-curve contains. Blyth and Kaka [19] produced a multiple linear regression model that predicts S-curves for individual projects, aiming at standardizing activities, and forecasting the duration, cost and end dates of these activities. Park [5] developed a cash flow forecasting model to help general contractors on a jobsite forecast cash flow during the construction stage. The model was based on the general procedure of construction jobsite and the nature of the general contractors' budget. Park et al.[20] adopted moving weights of cost categories in a budget that are variable depending on the progress of construction works aiming to provide a tool that can be applicable during the construction phase based on the planned earned value and the actual incurred cost on a job site level.
A second group of researchers focused on the factors affect project cash flow. Lowe [21] argued that the factors responsible for variation in project cash flow could be grouped under five headings of contractual, programming, pricing, valuation and economic factors. Harris and McCaffer [22] identified the factors that affect capital lock-up which ultimately affect project cash flow profile to include (profit margin, retention, claims, tender unbalancing, delay in receiving payments from clients and delay in paying labours, plant hires, materials' suppliers and subcontractors. Calvert [23] identified other factors to include seasonal effects on construction works, variability in preliminary expenses, contract extensions of time for inclement weather and valuation of variations.
A third group of researchers introduced optimization models for cash flow management. Karshenas and Haber [24] are among those who first introduced optimization models in cash flow management. Their model aimed at minimization of the total project cost through cash flow forecast. Barbosa and Pimentel [25] conducted significant research in proposing a linear programming model which is designed for optimal cash flow management addressing maximizing final cash balance. Their model included typical financial transactions, possible delays on payments, use of available credit lines, the effect of changing interest rates, and budget constraints that often occur in the construction industry. Elazouni and Gab-Allah [26] introduced an integer programming finance-based scheduling method to produce financially feasible schedules that balance the financing requirements of activities at any period with the cash available during the same period. The proposed method offers twofold benefits of minimizing total project duration and fulfilling finance availability. Liu and Wang [27] applied combinatorial optimization algorithms based on constraint programming to integrate the issues involving resource constrained problems and cash

flow. Also, Liu and Wang [28] presented a two stage profit optimization model for linear scheduling problems using constraint programming to optimize the primary objective: project profit and minimize total interruption time, given the optimized value of the primary objective. Liu and Wang [29] presented an optimization model considering cash flow for multi-project scheduling problems to determine schedules and periodical cash flow in an effort to maximize overall profit. Elazouni and Abido [30] presented a multi-objective optimization approach which can be used by lenders to make decisions regarding the fund allocation to the contractors based on the determination of the contractors' exact finance needs. The proposed fund allocation process fulfills the lenders' fund constraints and allows them to give priority to contractors of good record. The proposed model helps make decisions that minimize the financial risk born by the lenders and maximize the utilization of their fund.

In the field of artificial intelligence, Boussabaine and Kaka [31] have attempted to model cash flow forecast using artificial neural networks, which simulates neuronal systems of the brain. Boussabaine and Elhag [32] applied fuzzy set theory to model movement of cash flow at valuation periods. Attempts have also been made modeling cash flow forecast using expert systems. Efforts in this regard include that of Bandon [33]; Saleh [34]; Moussa [35]; Lowe et al.[36] and Lowe and Lowe [37]. While some of these expert system models focused on the construction contractors, others focused on the clients. Elazouni and Metwally [38] utilized genetic algorithm technique to device finance-based schedules that maximize project profit through minimizing financing cost and indirect costs. Finance-based scheduling provides a tool to control the credit requirements which enables the contractor to negotiate lower interest rates to reduce financing costs. Afshar and Fathi [39] presented a new approach to investigate multi-objective finance-based scheduling for construction projects under uncertainty. They developed a multi-objective model to search the non-dominated solutions considering total duration, required credit, and financing cost as three objectives. Fuzzy-sets theory is used to account for uncertainties in direct cost of each activity for determining the required credit and financing cost. The model fully embeds fuzzy presentation of the uncertainties in direct cost into the model structure. The α-cut approach is used to account for the accepted risk level of the project manager, for which a separate Pareto front with set of non-dominated solutions has been developed. El-Abbasy et al. [40] developed a multi-objective elitist non-dominated sorting genetic algorithm for solving finance-based scheduling problem of multi-projects with multi-mode activities. A critical path method scheduling model is constructed with its associated cash flow to calculate the values of the multiple objectives. The problem involves the minimization of conflicting objectives: duration of multiple projects, financing costs, and maximum negative cumulative cash balance. Alghazi et al. [41] used the Shuffled Frog-Leaping Algorithm (SFLA) to solve NP-hard combinatorial problem of finance-based scheduling. The performance of the SFLA was evaluated through benchmarking its results against those of Genetic Algorithm (GA) and Simulate Annealing (SA). The traditional problem of generating infeasible solutions in scheduling problems was adequately tackled in the implementations of the GA, SA, and SFLA. The results indicated that the SFLA improved the quality solutions with a substantial reduction in the computational time.

Recently, Kim and Kim [42] examined the sensitivity of the performance of seven project duration forecasting methods in the earned value management (EVM) literature to characteristic patterns of planned value and earned value S-curves. They identified the relative robustness and early warning capacity of six deterministic methods and one probabilistic method with respect to the nonlinearity of progress curves and the schedule delay patterns. The sensitivity analysis showed that forecast accuracy and early warning credibility of deterministic methods are very sensitive to the S-curve patterns, especially early in a project. The results revealed that the probabilistic method (the Kalman filter earned value method) is the only method among the seven alternatives that is robust with respect to the progress curve nonlinearity and the schedule delay patterns.

III. Fuzzy Linear Programming in Literature

Fuzzy linear programming (FLP) was recently applied as a new technique for handling optimization of multi objective problems. Raju and Kumar [43] developed a FLP model for the evaluation of management strategies of irrigation for a case study of Sri Ram Sagar project, Andhra Pradesh, India. Three conflicting objectives; net profits, crop production and labour employment were considered in the irrigation planning scenario. Kumar et al. [44] applied fuzzy linear programming in construction projects. They illustrated the practicability of applying fuzzy linear programming to civil engineering problem and the potential advantages of the resultant information. Trakiris and Spiliotis [45] applied FLP for problems of water allocation under uncertainty. In their work, a fuzzy set representation of the unit revenue of each use together with a fuzzy representation of each set of constraints, were used to expand the capabilities of the linear programming formulations. Eshwar and Kumar [46] used FLP to identify the optimum number of pieces of equipment required to complete the construction project in the targeted period with fuzzy data. Mohan and Jothi [47] used FLP for optimal crop planning for irrigation system dealing with the uncertainty and randomness for the various factors affecting the model. Cross and Cabello [48] applied fuzzy set theory to optimization problems, where multiple goals exist. They solved a multi-objective LP problem with fuzzy parameters for borrowing/lending problem. Faheem et al. [49] demonstrated the applicability of fuzzy linear programming for project least-cost scheduling. They presented a practical application of fuzzy linear programming in a real-life project network problem with two objectives. These objectives, were minimum completion time and crashing costs required to be optimized simultaneously. Regulwar and Gurav [50] developed a multi objective fuzzy linear programming approach for crop planning in command area of Jayakwadi project stage I, Maharashtra State, India. Four objectives were optimized (maximized) simultaneously. These objectives were the Net Benefits (NB), Crop/Yield Production (YP), Employment Generation (EG) and Manure Utilization (MU). However, literature review demonstrated that FLP has not been adopted for cash flow management for optimization purposes of the three above objectives. This paper presents a multi-objective fuzzy linear programming for cash flow management of construction projects by incorporating three objectives simultaneously; maximization of final cash balance, minimization cost of money, and minimization of initial cash.

IV. Multi Objective Fuzzy Linear Programming

Raju and Kumar [43] explained that fuzzy linear programming problem associates fuzzy input data by fuzzy membership functions. They added that FLP model assumes that objectives and constraints in an imprecise and uncertain situation can be represented by fuzzy sets. The fuzzy objective function can be maximized or minimized. In FLP the fuzziness of available resources is characterized by the membership function over the tolerance range (Raju and Kumar) [43]. However, in conventional LP, the problem is defined as follows (Zimmerman) [51]:

Maximize Z = CX (1)
Subject to AX ≤ B (2)
And X ≥ 0 (3)

In the fuzzy linear programming the problem can be restated as
Find X such that

CX ≤ Z (4)
AX ≤ B (5)
X ≥ 0 (6)

The membership function of the fuzzy set "decision model" [μD(X)] is given by Eq.7

μD(X) = mini {μi(X)}; i = 1, 2, n (7)

μi(X) can be interpreted as the degree to which X fulfils the fuzzy inequality CX ≤ Z and n is the number of objective functions. In the planning scenario, decision maker is not interested in a fuzzy set but in crisp optimum solution, maximizing Eq.7 gives Eq.8.

MaxX≥0 μD(X) = MaxX≥0 mini {μi(X)} (8)

Membership function μi(X) is represented as

$$\mu i(X) = 0 \quad \text{for } Z \le ZL$$
$$= \frac{Z - Z_L}{Z_U - Z_L} \quad \text{for } ZL < Z < ZU \qquad (9)$$
$$= 1 \quad \text{for } Z \ge ZU$$

ZU = Aspired level of objective
ZL = Lowest acceptable level of objective

μi(X) reflects the degree of achievement. Value of μi(X) will be 1 for perfect achievement and 0 for no achievement (worst achievement) of a given strategy and some intermediate values otherwise. The model can be transformed as follows:

$$\text{MaxX}\ge 0 \text{ mini } \frac{Z - Z_L}{Z_U - Z_L} \qquad (10)$$

Subject to:

AX ≤ B (11)
X ≥ 0 (12)

Introducing a new variable λ, the FLP problem can be formulated as equivalent LP model.

Max λ
Subjected to:

$$\frac{Z - Z_L}{Z_U - Z_L} \ge \lambda \qquad (13)$$

For each objective

AX ≤ B (14)
0 ≤ λ ≤ 1 (15)
X ≥ 0 (16)

and all the exiting constraints:
Briefly the FLP algorithm is divided into six steps:

1. Solve the problem as a linear programming problem by taking only one of the objectives at a time.
2. From the results of step 1, determine the corresponding values of every objective at each solution derived.
3. From step 2, best (ZU) and worst (ZL) values can be calculated.
4. Formulate the linear membership function.
5. Formulate the equivalent linear programming model for the fuzzy multi objective.
6. Determine the compromise solution along with degree of truth (λ).

V. Jiang et al. Model (2011)

Jiang et al.[4] presented a Pareto optimality efficiency network model aimed to maximize final cash balance (FC) and minimize total cost of money

(*R*) by determining such variables as the long term loan (*LTL*) and the periodic short term loans (STLi) for a project. There were some pre-defined external inputs to the network such as the periodic expense forecasts (*Ei*), owner's progress payment (Pi) as defined by the payment plan, and front money as the initial capital (*IC*). Also, pre-defined parameters included retainage rate (r_4), profit percentage (r_5), periodic minimum cash balance requirement (*V*), and all kinds of interest rates associated with the cost of money from long term loan and short-term loans, and related to earnings from excess cash balance. In their model, they presented the equations given next. Eq.(s) 17 and 18 represent the two objective functions, which are maximizing the final cash balance at the end of period *n*+1 and minimizing the total cost of money. In Eq.18, *Ri* is the interests paid to the banks or cost of money at the end of period i or at the beginning of period *i* + 1. *LTL* is the long term loan issued to the project when the project starts. The interest of *LTL* is paid periodically and the principle should be paid off when the project finishes at the end of period . STL_i is the short term loan cash flow at the beginning of the period . STL_i and its interests should be paid off at the end of period or at the beginning of the period *i*+1. Otherwise, the contractor is not entitled to further short term loans or is charged additionally depending on existing terms between the bank and the contractor. It must be noted that the computation of cost of money R_{n+1} in Eq.17 is different from R_i (see Eq.20) in a typical period. The first contains only STL_{n+1} and its interest as in Eq.24, but the latter contains the long and short term loans (Eq.23).

Beginning of first period is the beginning of the project. It has three cash inflows: initial capital (*IC*), long term loan (*LTL*), and short term loan (STL_1) and one cash outflow, which is the first periodic project expense (E_1) (see Eq.19). CB_1 is the mathematical sum of all cash inflows and outflows at beginning of first period. Initial capital or front money is required which is assumed as available at the beginning of the planning horizon. STL_1 is the short term loan issued at the beginning of the project. *LTL* can be a construction loan or other kinds of loans. It represents a developer's or a contractor's borrowing capacity. It is assumed that *LTL* is only available at the beginning of the planning horizon. Long term loans in this model are supposed to be paid off at the completion of the project at end of period n. In other words, end of period is the point in time to pay the principle of *LTL* back to the banks. Therefore, there is one more cash outflow at end of period n comparing to end of any other period *i*.

Eq.(s) 19 to 26 represents the group of constraints.

$$\text{Max } FC = G + P_{n+1} + CB'_{n+1} - STL_{n+1} - R_{n+1} \tag{17}$$

$$\text{Min } R = \sum_{i=1}^{n+1} R_i = \sum_{i=1}^{n} (LTL \times r_2)_i + \sum_{i=1}^{n+1} STL_i r_3 \tag{18}$$

Subject to:

$$CB_1 = STL_1 + LTL + IC - E_1$$

(for end of period =) (19)

$$CB_{i+1} = STL_{i+1} + P_i + CB'_i - E_{i+1} - STL_i - R_i$$

(for end of period = , ,… ..,) (20)

$$CB_{n+1} = STL_{n+1} + P_n + CB'_n - E_{n+1} - STL_n - R_n$$

(for end of period =) (21)

$$CB'_i = (1 + r_1)CB_i$$

(for end of period i = 1, 2, ...i..., n-1) (22)

$$R_i = (LTL)r_2 + (STL_{i+1})r_3$$

(for end of period i = 1, 2, ..., n) (23)

$$R_{n+1} = (STL_{n+1})r_3 \tag{24}$$

$$G = \sum_{i=1}^{n} r_4 \times E_i \times (1 + r_5) \tag{25}$$

$$CB_i > V$$

(for end of period i = 1, 2, ... n + 1) (26)

Where: *G* is the total money retained by the owner; P_{n+1} is the owner's full payment for the project which occurs in the period *n*+1; CB_{n+1} is the cash balance at the end of period *n*+1; STL_{n+1} is the short term loan in the period *n*+1; R_{n+1} is the cost of money at the end of period *n*+1; *Ri* is the periodic cost of money paid to the banks; r_2 is the interest rate for long term loan; r_3 is the interest rate for short term loan; STL_1 is the short term loan issued at the beginning of the project; E_1 is the first periodic project expense; CB_{i+1} is the cash balance at the beginning of period *i*+1; CB_i is the periodic cash balance at the end of period i and which considered cash inflow for period *i*+1; *Ei+1* is the project expense for period *i*+1; *CBn*+1 is the cash balance at the beginning of period *n* +1. P_n is the owner's payment for the project expense which occurs in the period n; CB_n is the cash balance at the end of period n; E_{n+1} is the project expense for period *n*+1; STL_n is the short term loan in the period n, Rn is the cost of money at the end of period *n*, r_1 is the interest rate for excess cash deposited.

In their model, Jiang et al. [4] reported that cash forecasts and project parameters are used as input to the model. Once they were defined, the optimality efficiency algorithm served as an analytical tool for various scenarios by changing the project parameters and financial constraints (e.g. front money, minimal periodic cash balance, etc.) to manipulate the cash transactions over the planning horizon, aiming at achieving a greater profitability and less cost of money level for the project. Jiang et al. [4] explained that two pairs of values on the cost of money and final cash balance are obtained by minimizing the cost of money and maximizing the final cash balance. The first pair (FC_R , R_{min}) is the total cost of money (R_{min}) and the final cash balance (FC_R) by minimizing the objective function R. The second pair is the total cost of money(R_{FC}) and the final cash balance (FC_{max}) by maximizing the objective function FC . The value ranges of the cost of money and final cash balance are (R_{min}, R_{FC}) and (FC_R, FC_{max}) (see Fig.1). Given the various R within the range of (R_{FC}, R_{min}) as an upper limit of the cost of money (constrain), the maximal values of FC are found by running the network model. If all optimal solutions are graphed in the x-y plane with the y-axis being the values on Objective 1 (maximizing final cash balance) and the x-axis being the values on Objective 2 (minimizing interest paid), the graph is called a trade-off curve or efficient frontier. They added that, for illustration, suppose that the set of feasible solutions for the bi-objective problem is the shaded region bounded by the curve AB and the first quadrant in Fig. 1, then the curve AB is the set of Pareto optimal points under pre-defined parameters and external inputs. They further gave the steps for finding a Pareto optimality trade-off curve.

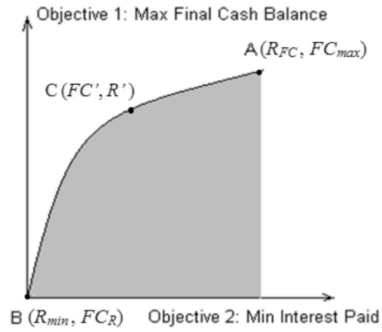

Fig. 1: Pareto optimality trade-off curve in max-min problems (jiang et. al, 2011).

VI. Proposed Model Development

The proposed model is a modification for Jiang et al. model [4] in the following aspects:

1. The objective function maximize the final cash balance (FC) presented in Eq.17 is modified to maximize the final cash balance considering both advanced payment and delay of payment (FC_{AD}) as in Eq.27. It must be noted that a penalty percentage (PP) on delayed payment will be considered. This penalty is paid by the owner to the contractor. Eq.(s) 28 and 29 represent the final cash balance in the case of advanced payment only (FC_A) and delay of payment only (FC_D), respectively.

$$\text{Max} \quad FC_{AD} = G + P_{n+2} + CB'_{n+2} - STL_{n+2} - R_{n+2} - LP_{n+1} \times APP + PP \times P_{n+2} \qquad (27)$$

$$\text{Max} \quad FC_{A} = G + P_{n+1} + CB'_{n+1} - STL_{n+1} - R_{n+1} - LP_{n+1} \times APP \qquad (28)$$

$$\text{Max} \quad FC_{D} = G + P_{n+2} + CB'_{n+2} - STL_{n+2} - R_{n+2} + PP \times P_{n+2} \qquad (29)$$

Where: P_{n+2}, $CBn+2$, $STLn+2$, and R_{n+2} are as defined previously but for the period n+2 due to delay one period; LP_{n+1} is the last payment before the retainage and which occur in period n+1; APP is the advanced payment percentage; PP is the penalty percentage .

2. The objective function minimize the cost of money (R) presented in Eq.18 is modified to minimize the cost of money considering both advanced payment and delay one period for the owner's progress payment simultaneously or considering delay only ($R_{AD,\,D}$) as in Eq.30, otherwise Eq.18 is used. Accordingly, Eq.(s) 31 and 32 corresponds to Eq.(s).23 and 24.

Min

$$R_{AD,D} = \sum_{i=1}^{n+2} R_i = \sum_{i=1}^{n+1} (LTL \times r_2)_i + \sum_{i=1}^{n+2} STL_i r_3 \qquad (30)$$

$$R_i = (LTL) r_2 + (STL_{i+1}) r_3$$

(for end of period i = 1, 2, ... n+1) (31)

$$R_{n+2} = (STL_{n+2}) r_3 \qquad (32)$$

3. An additional objective which is to minimize the initial cash balance (IC) is considered in Eq.33. Also a constraint for IC is added as given in Eq.34. In Eq.34 in which MPi is the periodic monthly payment before cutting the retainage rate, L is the percentage produced from the optimization process. Eq. 35 gives a constraint for the maximum allowed percentage of initial cash balance with respect to the contract value (IC_P). This percentage is decided by the model's user.

Min IC (33)

$$IC \le \left(\sum_{i=1}^{n} MP_i\right) \times L \qquad (34)$$

$$L \le IC_P \qquad (35)$$

4. In the new formulation, the mathematical sum of all cash inflows and outflows at the beginning of first period considering both advanced payment (AP) and delay one period for the owner's progress payment simultaneously or advanced payment only (CB_{APD}, AP)1 is represented by Eq.36, otherwise

Eq.19 is used. Also, Eq.(s) 37 and 38 are used for considering both an advanced payment and delay of payment simultaneously instead of Eq.20. Considering advanced payment only Eq.39 is applied. Considering delay of payment only Eq.(s) 37 and 40 are used. Eq.41 is used in case of considering advanced payment and delay simultaneously or delay only, otherwise Eq.26 is used. Eq.25 is applied for any case.

$$(CB_{APD,AP})_1 = STL_1 + LTL + IC - E_1 + AP$$

(for end of period i = 0 or beginning of first period) (36)

$$(CB_{APD})_2 = STL_2 + CB'_1 - E_2 - STL_1 - R_1$$

(37)

$$(CB_{APD})_{i+1} = STL_{i+1} + P_{i-1}(1+PP) - MP_{i-1} \times APP + CB'_i - E_{i+1} - STL_i - R_i$$

(for beginning of period I = 2, ...n) (38)

$$(CB_{AP})_{i+1} = STL_{i+1} + P_i - MP_i \times APP + CB'_i - E_{i+1} - STL_i - R_i$$

(for beginning of period i = 1, ... n) (39)

$$(CB_D)_{i+1} = STL_{i+1} + P_{i-1}(1+PP) + CB'_i - E_{i+1} - STL_i - R_i$$

(for beginning of period i = 2, ... n) (40)

$$(CB_{APD,D})_i > V$$

(for end of period i = 1, 2, ... n+2) (41)

Where $(CB_{APD})_2$ is the sum of all cash inflows and outflows at the beginning of second period considering advanced payment and delay; $(CB_{APD})_{i+1}$ as $(CB_{APD})_2$ but for beginning of period i+1; $(CB_{AP})_{i+1}$ is the sum of all cash inflows and outflows at the beginning of period i+1 considering advanced payment only; $(CB_D)i$+1 is the sum of all cash inflows and outflows at the beginning of period i+1 considering delay only and $(CB_{APD,\ D})_i$ is the sum of all cash inflows and outflows at the beginning of period considering advanced payment and delay or delay only.

VII. Model Implementation

The proposed model developed above is applied to an example project given by Liu et al. [1]. The data belongs to a building located in Tianjian, China, with estimated cost 6, 570, 059 Yuan and a 7.1% profit margin (r5). The project lasted five months and its data given in Table 1(columns 1, 2, and 3). Data presented in columns 4 and 5 are calculated by the author. The contractor received the payment from the owner on a monthly basis according to the percentage of the project that had been completed. The following monthly interest rates are assumed and adopted: interest rate for excess cash deposited r_1 = 1.25%, interest rate for long term loan r_2 = 6%, interest rate for short term loan r_3 = 7.5% and retainage rate r_4 = 10%. The owner's payment is the monthly payment after cutting 10% retainge rate (see Table 1). Also, 50% of monthly payment is assumed as a minimal cash balance requirement as given in Table 1. In addition, two values for the advanced payment 5% and 10% are assumed and adopted in solving procedure.

7.1 Solving procedure

An individual optimization for each objective will be performed and a comparison of solutions will then be presented through the following subsections, four cases are adopted. These are: neglecting advanced payment and delay of payment, considering advanced payment only, considering delay of payment, and considering advanced payment and delay of payment simultaneously. On the other hand, since two values for advanced payment: 5% and 10% are adopted, thus a total of 6 cases are considered as given in Table 2.

7.2 Individual optimization

An optimization of each individual objective: maximizing final cash balance, minimizing cost of money, and minimizing initial cash balance is performed with linear programming software (Lindo). The objectives are conflict with one another. Thus, there is a need to strike a balance and develop a tradeoff relationship between maximizing final cash balance, minimizing the cost of money, and minimizing initial cash balance. The goal is to select a compromise alternative to meet the chosen levels of satisfaction as would be demanded in the decision making process. The upper and lower bounds for the multi-objective analysis was obtained and presented in Table 2. Ideal and worst values are denoted with an asterisk and plus, respectively.

Table1: Input data for the example project

Month (1)	Contractor's Expenses (2)	Monthly payment (3)	Owner's Payment (3)×90% (4)	Minimal cash balance require. (V) = (2) × 50%= (5)
1	2496622	0	0	1248311
2	1576814	2675647	2408082	788407
3	1511114	1689882	1520894	755557
4	657006	1619470	1457523	328503
5	328503	704118	633706	164252
6	0	352059	176030+704118	0
Total	6570059	7041176	7041176	--

7.3 Multi-objective fuzzy linear programming

Since the objective is to maximize final cash balance, minimize the cost of money, and minimize initial cash balance simultaneously, best values (Z_U) will be the maximum values obtained in individual optimization process for the objective maximizing final cash balance and Eq.13 is applied. But for the other two objectives the worst values (Z_L) will be the maximum values obtained in individual optimization process. Also, Eq.13 will become as presented in Eq.42. The complete formulation for the example when considering 5% advanced payment and delay of payment simultaneously (for example) represented by Eq.(s) 43-45, and all the exiting constraints.

$$\frac{Z - Z_L}{Z_U - Z_L} \leq \lambda \tag{42}$$

Max λ subjected to:

$$\frac{FC - 800000}{1338970 - 800000} \geq \lambda \tag{43}$$

$$\frac{R - 1096077}{797203 - 1096077} \leq \lambda \tag{44}$$

$$\frac{IC - 1019774}{zero - 1019774} \leq \lambda \tag{45}$$

Table 2: Ideal values for individual optimization and three objectives FLP

Case	Objective	Individual optimization Max . FC	Individual optimization Min. R	Individual optimization Min. IC	Three objectives FLP
Considering 5% advanced payment and delay of payment	Final cash balance Cost of money Initial capital	1338970* 1093939 0*	800000+ 797203* 1019774+	1172666 1096077+ 0*	1081281 940099 487567
	Associated (λ)				0.52
	Objective	**Max . FC**	**Min. R**	**Min. IC**	**Three objectives FLP**
Considering10% advanced payment and delay of payment	Final cash balance Cost of money Initial capital	1321190 300000 3170207	1321190 278710 3000000	1321190 1310320 0	1321190 600000 1095906
	Associated (λ)				0.65
	Objective	**Max . FC**	**Min. R**	**Min. IC**	**Three objectives FLP**
Considering 5 % advanced payment	Final cash balance Cost of money Initial capital	1335977* 221666* 1043410+	1335977 221666* 1043410+	1335977* 598974 0*	1335977 383907 448664
	Associated (λ)				0.57
	Objective	**Max . FC**	**Min. R**	**Min. IC**	**Three objectives FLP**
Considering 10% advanced payment	Final cash balance Cost of money Initial capital	1318374* 196297* 1000437+	1318374* 196297* 1000437+	1318374* 469206+ 0*	1318374 330623 492417
	Associated (λ)				0.51
Case	**Objective**	**Individual optimization**			**Three**

		Max. FC	**Min. R**	**Min. IC**	**objectives FLP**
Considering delay of payment	Final cash balance Cost of money Initial capital	1356748* 1165523 0.0*	900000^{+} 815327* 1137724^{+}	1190444 1167661^{+} 0*	1136798 984996 547878
	Associated (λ)				0.52
Neglecting advanced payment and delay of payment	**Objective**	**Individual optimization**			**Three objectives FLP**
		Max. FC	**Min. R**	**Min. IC**	
	Final cash balance Cost of money Initial capital	1353580* 247036* 1086383^{+}	1353580* 247036* 1086383^{+}	1353580* 543391^{+} 0*	1353580 395213 543191
	Associated (λ)				0.50

Results for the optimum values for the three objectives when applying FLP are presented in Table 2. In this Table, it can be shown that the degree of truth (λ) ranges from 0.5 to 0.65 for optimizing the three objectives. The optimum value of initial cash balance in crisp LP (individual optimization) is zero in all cases. Also, it can be shown that the maximum percentage of initial cash balance required in FLP is approximately 15.6% of contract value.

Table 3 shows the deviation of the three objectives FLP as compared to ideal values in the crisp linear programming (LP) model for different cases. It can be shown that the optimum value of final cash balance reduced by a percentage ranges from zero to 19% in FLP from the corresponding ideal value in the crisp linear programming (LP) model. On the other hand, the cost of money conflict highly with final cash balance, thus two categories for cost of money can be dealt with. In the first category, the optimum value of R increased by a large percentage ranges from 59.9% to 115% in FLP from corresponding ideal value in the crisp LP due to constant values of FC in both FLP and LP. In the second category, the optimum value of R increased by a small percentage 17.9% to 20.8% relative to first category in FLP from the corresponding ideal value in crisp LP due to the ideal value of in FLP from the corresponding ideal value in crisp LP. Also, the optimum value of initial cash balance increased by a percentage approximately ranges from 6.4% to 15.6% in FLP from the corresponding ideal value in the crisp LP.

Table 3: Deviation (percentage or value) of the three objectives FLP from crisp LP

Case	**Advanced payment percentage**	**% Deviation**		***IC***	
		FC	***R***	**Increasing value**	**% of contract value**
Considering advanced payment and delay of payment	5%	19%	17.9%	487567 Yuan	6.9%
	10%	Zero	115 %	1095906 Yaun	15.56%
Considering advanced payment	5%	Zero	73%	448664 Yaun	6.37%
	10%	Zero	68%	492417 Yaun	7%
Considering delay of payment	------	16.2%	20.8%	547878 Yaun	7.78%
Neglecting advanced payment and delay of payment	------	Zero	59.9%	543191 Yaun	7.7%

VIII. Conclusions

The multi-objective fuzzy linear programming model presented in this paper is aimed at providing cash flow management for projects in the tendering and construction stages. The proposed model resolving the optimization problem of three conflicting objectives: final cash balance, cost of money, and initial cash balance. The proposed model depends on Jiang et al. (2011) model which considered the constraints of the financial market, the budget constraint, and retention of money. In addition to these variables, in the proposed model, both the advanced payment and delay of owner's progress payment one period have been represented separately and simultaneously. Penalty on delayed payment have been also considered. The proposed model was validated using data of an example project. An individual optimization for each objective was performed separately with linear programming software (Lindo) that gave the upper and lower bounds for the multi-objective analysis.

Examining results of the example project revealed that; (1) fuzzy linear programming is simple and suitable tool for multi-objective problems and (2) the model can be extended to any number of objectives by incorporating only one additional constraint in the constraint set for each additional objective function. On the other hand, the model enables contractors to generate and evaluate all optimal tradeoff solutions between any two objectives; final cash balance and cost of money; final cash balance and initial cash balance; or cost of money and initial cash balance that suit their ordering of preferences and demands. Although the model considers a good deal of variables and trade-off decision objectives, it is still a limited representation of the complex real world of the construction management environment. An example of the other factors is the delay of the client's progress payment more than one period. Also, more decision objectives may become additional concerns in the decision making for this full-of-uncertainty industry. Finally, the model presents an effective decision making tool to be used by industry practitioners with reasonable accuracy.

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